MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 2, 2018/2019

EMT1026 – ENGINEERING MATHEMATICS 2

(All Sections / Groups)

4 MARCH 2019

9.00 AM – 11.00 AM

(2 Hours)

INSTRUCTIONS TO STUDENTS:

1. This exam paper consists of **10 pages** (including cover page) with **4** Questions only.
2. Attempt **all** 4 questions. All questions carry equal marks and the distribution of marks for each question is given.
3. Please print all your answers in the Answer Booklet provided. Show all relevant steps to obtain maximum marks.
4. Several tables are provided in the **Appendix** for your reference.

Question 1

(a) Verify that $y=e^{-x}$ and $y=e^{3x}$ are the solutions of the homogeneous differential equation $y''-2y'-3y=0$. Hence solve the non-homogeneous equation

$$y''-2y'-3y=3x^2+4x-5.$$

[12 marks]

(b) Consider the second order differential equation $\left(1+x^2\right)y''-3xy'-5y=0$.

Using power series solution $y=\sum_{n=0}^{\infty}c_nx^n$, prove that $c_2=\frac{5}{2}c_0$, $c_3=\frac{4}{3}c_1$ and the recurrence relation is given by $c_{k+2}=\frac{\left[3k+5-k\left(k-1\right)\right]}{\left(k+2\right)\left(k+1\right)}c_k$, $k=2,3,4,\ldots$.

Hence, find the first five terms of the power series solution.

[13 marks]

Question 2

(**Note**: The tables in APPENDIX A and APPENDIX B may be useful for solving this question.)

(a) Find the Laplace transform of:

(i) $f\left(t\right)=4t^2e^{-3t}$ by using the s-shifting property. [3 marks]

(ii) $f\left(t\right)*g\left(t\right)$, given that $f\left(t\right)=\delta\left(t-\frac{\pi}{2}\right)$ and $g\left(t\right)=\sin 3t$. [4 marks]

(b) Find the inverse Laplace transform of the following:

(i) $F\left(s\right)=\frac{e^{-3s}}{s^2-4}$ by using the t-shifting property. [3 marks]

(ii) $-\frac{d}{ds}\left(\frac{2s}{s^2+64}\right)$ by using the differentiation of transform property.

[3 marks]

(c) Solve the following differential equation by using Laplace transform method:

$$y''\left(t\right)-5y'\left(t\right)+6y\left(t\right)=e^{4t}$$

subject to $y\left(0\right)=1, y'\left(0\right)=0$. [12 marks]

Continued...

Question 3

(a) Consider a thin, homogeneous bar of length L with both ends insulated. The initial temperature of the bar is given by $f(x)$ and the bar is placed in a medium of temperature T. The equation governing the temperature distribution along the bar at time t, $u(x,t)$, is given by

$$\frac{\partial u}{\partial t} = a^2 \frac{\partial^2 u}{\partial x^2} - A\left[u(x,t) - T\right],$$

where a is the thermal diffusivity and A is a constant. By setting $v(x,t) = e^{At}\left[u(x,t) - T\right]$, show that the above problem may be transformed into the heat equation:

$$\frac{\partial v}{\partial t} = a^2 \frac{\partial^2 v}{\partial x^2}.$$

Furthermore, deduce the corresponding boundary and initial conditions. DO NOT solve the problem.

[10 marks]

(b) Consider the following boundary value problem:

$$\frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2}, \qquad 0 \le x \le 1, \quad t \ge 0,$$

$$u(0,t) = u(1,t) = 0.$$

By using method of separation of variables, find the general solution of the above problem.

[15 marks]

Continued...

Question 4

(a) A continuous random variable, X, has probability density function

$$f(x)=\begin{cases} k(x+2), & -2 \le x \le 2, \\ 0, & \text{otherwise.} \end{cases}$$

(i) Show that $k = 1/8$. [3 marks]

(ii) Evaluate the mean of X. What are the chances that X exceeds its mean? [5 marks]

(**Note**: For solving Q4(b) and Q4(c) below, you may use the tables in APPENDIX D, APPENDIX E and APPENDIX F.)

(b) The weight of parcels handled by a certain courier company is normally distributed with mean value 5.5 kg and standard deviation 1.6 kg. Parcels up to 2 kg are classified as "Light Packets." Those exceeding 2 kg up to c kg are "Standard Packets," while the rest are "Heavy Packets."

(i) If $c = 9$, estimate the proportion of "Standard Packets" that are shipped by the company. [5 marks]

(ii) The company plans to impose a fuel surcharge on the "Heavy Packets." Determine the value of c such that 99% of parcels are unaffected by the surcharge. [4 marks]

(c) In the mass production of a certain electronic component, p% of the components are defective. Components are packed in boxes of 25 components/box.

(i) If p = 5%, determine the probability that a randomly selected box contains defective components. [4 marks]

(ii) Suppose when eight boxes were inspected, they were found to contain the following number of defectives:

	Box 1	**Box 2**	**Box 3**	**Box 4**	**Box 5**	**Box 6**	**Box 7**	**Box 8**
No. of defectives	0	1	2	0	1	0	0	2

Estimate p, then estimate the proportion of boxes that contain more than 2 defective components.

[4 marks]

End of questions.

APPENDIX A:

Common Laplace Transform Pairs

	$f(t)$	$F(s)=L\{f(t)\}$
1.	1	$\frac{1}{s}$
2.	t	$\frac{1}{s^2}$
3.	t^n	$\frac{n!}{s^{n+1}},\ n=1,2,\ldots$
4.	e^{at}	$\frac{1}{s-a}$
5.	$t^{n-1}e^{at}$	$\frac{(n-1)!}{(s-a)^n},\ n=1,2,\ldots$
6.	$\cos at$	$\frac{s}{s^2+a^2}$
7.	$\sin at$	$\frac{a}{s^2+a^2}$
8.	$\cosh at$	$\frac{s}{s^2-a^2}$
9.	$\sinh at$	$\frac{a}{s^2-a^2}$
10.	$u(t-a)$	$\frac{e^{-as}}{s}$
11.	$\delta(t-a)$	e^{-as}

Continued...

APPENDIX B:

Laplace Transform Properties

	Property Name	Formula
1.	Linearity	$L\{af(t)+bg(t)\}=aL\{f(t)\}+bL\{g(t)\}$
2.	s - shifting	$L\{e^{at}f(t)\}=F(s-a)$
3.	Transform of Derivative	$L\{f'(t)\}=sL\{f(t)\}-f(0)$ $L\{f''(t)\}=s^2L\{f(t)\}-sf(0)-f'(0)$
4.	Transform of Integration	$L\left\{\int_0^t f(\tau)d\tau\right\}=\frac{1}{s}F(s)$
5.	t - shifting	$L\{u(t-a)f(t-a)\}=e^{-as}F(s)$
6.	Differentiation of Transform	$L\{t.f(t)\}=-F'(s)$
7.	Integration of Transform	$L\left\{\frac{f(t)}{t}\right\}=\int_s^\infty F(\tilde{s})d\tilde{s}$
8.	Convolution Theorem	$L\{f(t)*g(t)\}=F(s).G(s)$, where $f(t)*g(t)=\int_0^t f(\tau)g(t-\tau)d\tau$.

Continued...

APPENDIX C

Half-range sine series:

$$f(x)=\sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{L}\right), \quad \text{where } b_n=\frac{2}{L}\int_0^L f(x)\sin\left(\frac{n\pi x}{L}\right)dx$$

Half-range cosine series:

$$f(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{L}\right), \quad \text{where } \mathrm{a}_0=\frac{2}{L}\int_0^L f(x)dx, \;\; a_n=\frac{2}{L}\int_0^L f(x)\cos\left(\frac{n\pi x}{L}\right)dx$$

Continued...

APPENDIX D:

Special Discrete Probability Distributions

Binomial Distribution, $X \sim b(x;n,p)$	
P.m.f.	$P(X=x) = {}^nC_x p^x q^{n-x}$, for $x = 0,1,\ldots,n$ and where $q = 1-p$.
Mean	$E[X] = np$
Variance	$Var(X) = npq$
Hypergeometric Distribution, $X \sim h(x;N,n,k)$	
P.m.f.	$P(X=x) = \frac{{}^kC_x \times {}^{N-k}C_{n-x}}{{}^NC_n}$, for $x = 0,1,\ldots\min(n,k)$.
Mean	$E[X] = \frac{nk}{N}$
Variance	$Var(X) = \frac{nk}{N}\left(\frac{N-n}{N-1}\right)\left(1-\frac{k}{N}\right)$
Poisson Distribution, $X \sim p(x;\lambda)$	
P.m.f.	$P(X=x) = \frac{\lambda^x e^{-\lambda}}{x!}$, for $x = 0,1,\ldots$.
Mean	$E[X] = \lambda$
Variance	$Var(X) = \lambda$

Continued...

APPENDIX E:

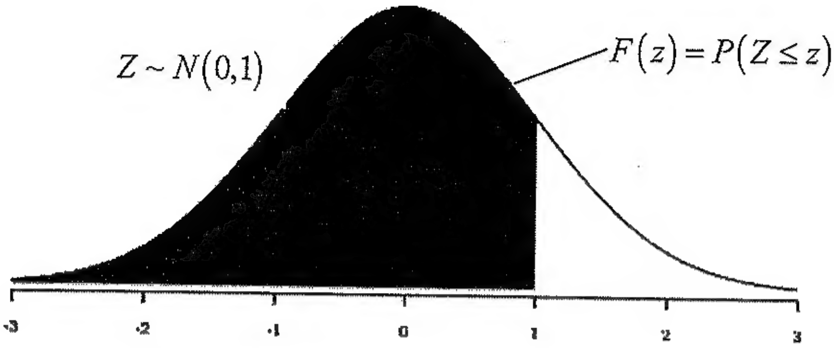
Special Continuous Probability Distributions

Continuous Uniform Distribution, $X \sim U(a,b)$	
P.d.f.	$f_X(x)=\begin{cases}\dfrac{1}{b-a}, & a<x<b\\ 0, & \text{elsewhere}\end{cases}$
Mean	$E[X]=\dfrac{a+b}{2}$
Variance	$Var(X)=\dfrac{(b-a)^2}{12}$
Exponential Distribution, $X \sim \exp(1/\beta)$	
P.d.f.	$f_X(x)=\begin{cases}\dfrac{1}{\beta}e^{-x/\beta}, & x>0\\ 0, & \text{elsewhere}\end{cases}$
Mean	$E[X]=\beta$
Variance	$Var(X)=\beta^2$
Normal Distribution, $X \sim N(\mu,\sigma)$	
If X is any normal random variable where $X \sim N(\mu,\sigma)$, then the transformation $$Z=\frac{X-\mu}{\sigma}$$ yields a *standard* normal variable where $Z \sim N(0,1)$.	

Continued…

APPENDIX F:

Cumulative Standard Normal Distribution



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990

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